

# Comment on Limitations on the superposition principle: superselection rules in non-relativistic quantum mechanics

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**Abstract.** This is a comment to the paper, Limitations on the superposition principle: superselection rules in non-relativistic quantum mechanics by C Cisneros et al 1998 Eur. J. Phys. 19 237. doi:10.1088/0143-0807/19/3/005.

The proof that the authors construct for the limitation on the superposition of state vectors corresponding to different sectors of the Hilbert space, partitioned by a superoperator has a flaw as outlined below.

## 1. Introduction

In Ref. [1], section 2.4 (Impossibility of superposing states belonging to different coherent sectors), the authors construct a proof using a superoperator  $G$  that commutes with all observables of the system. The eigenstates of operator  $G$  are of the form  $|g_m; \alpha_m\rangle$  where  $g_m$  and  $\alpha_m$  are labels to distinguish different eigenvectors all of which are non-degenerate, i.e.  $\langle g_m; \alpha_m | g_n; \alpha_n \rangle = 0$  if  $m \neq n$ .

In the next section (2.4), they construct a vector  $|u\rangle = \sum_m u_m |g_m; \alpha_m\rangle$  superposing the eigenstates of  $G$  and claim that since  $G$  commutes with every other observable,  $|u\rangle$  should be an eigenstate of every operator in a complete set of commuting observables of the system.

Now we know that if  $G|a\rangle = a|a\rangle$  and  $G|b\rangle = b|b\rangle$  are eigenvectors of an operator  $G$ , then  $\lambda_1|a\rangle + \lambda_2|b\rangle$  is not an eigenvector unless they have the same eigenvalue since:

$$G(\lambda_1|a\rangle + \lambda_2|b\rangle) = a\lambda_1|a\rangle + b\lambda_2|b\rangle \neq \lambda(\lambda_1|a\rangle + \lambda_2|b\rangle) \quad (1)$$

(for some  $\lambda$ ) unless  $a = b$ , which is not true in general and definitely not true for the hermitian operators with non-degenerate eigenvalues in the paper.

It is true that if  $G$  commutes with every other observable and has non-degenerate eigenvalues, then all such observables share the same eigenvectors. But  $|u\rangle$  itself is not an eigenvector of  $G$  as proved above. And so  $|u\rangle$  cannot be an eigenvector of other observables too. To summarize, the proof of superselection rules proposed is thus is not valid for the case discussed by the authors. Whether a similar proof can be constructed is still open.

## References

- [1] C Cisneros et al 1998 *Eur. J. Phys* 19 237, doi:10.1088/0143-0807/19/3/005, Limitations on the superposition principle: superselection rules in non-relativistic quantum mechanics.